7-4 Lengths of Curves

Learning Objectives:

I can find the length of smooth curve

I can find the length of curve that has vertical tangent lines, corners, or cusps.

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Length of a Smooth Curve

If a smooth curve begins at (a,c) and ends at (b,d), where a < b, c < d, then the length of the curve (arc length) is:

$$\int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$

if y is a function of x

$$\int_c^d \sqrt{1+\left(\frac{dx}{dy}\right)^2}\,dy$$

if x is a function of y

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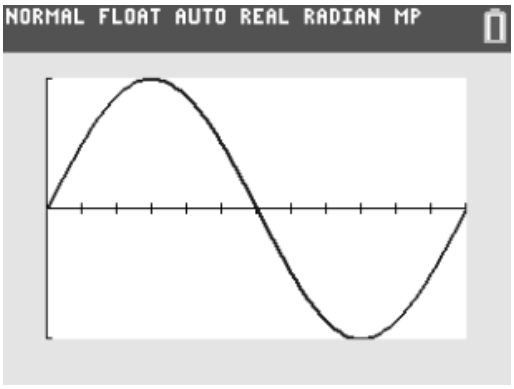
Ex1. Find the length of the curve

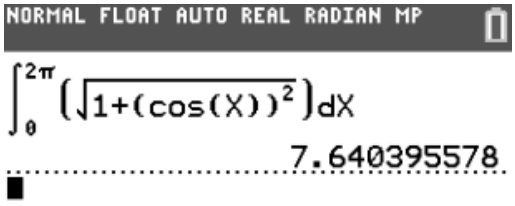
1.) y = sin(x) on $[0, 2\pi]$

$$\int_a^b \sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx$$

$$\frac{dy}{dx} = \cos x$$

$$\int_0^{2\pi} \sqrt{1+(\cos x)^2}\,dx$$





2.) $y = 3e^{-1/2x}$ on [0, 1]

$$\int_0^1 \sqrt{1+\left(\frac{-3}{2}e^{-.5x}\right)^2}\,dx = 1.551$$

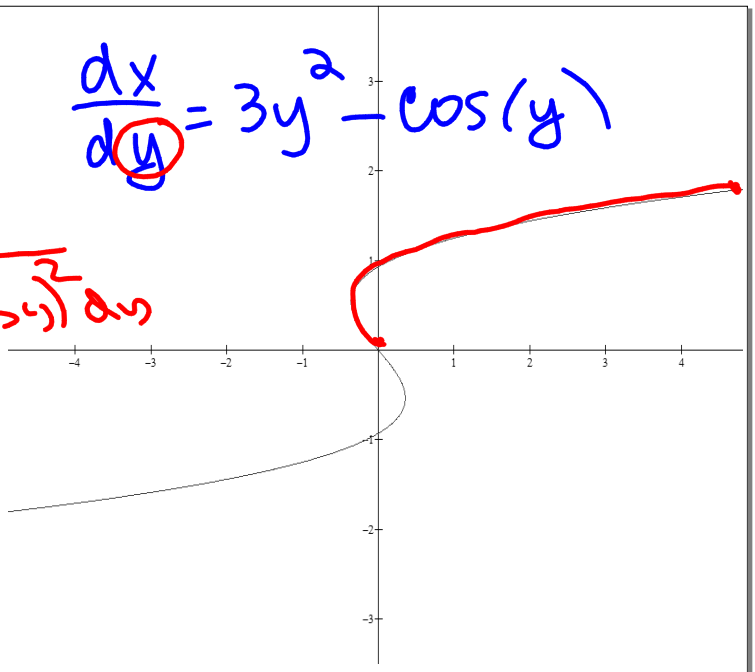
3.) $x = y^3 - \sin(y)$

from y = 0 to $y = \frac{\pi}{2}$

$\frac{dx}{dy} = 3y^2 - \cos(y)$

$\int_0^{\frac{\pi}{2}} \sqrt{1 + (3y^2 - \cos y)^2}\, dy$

4.173



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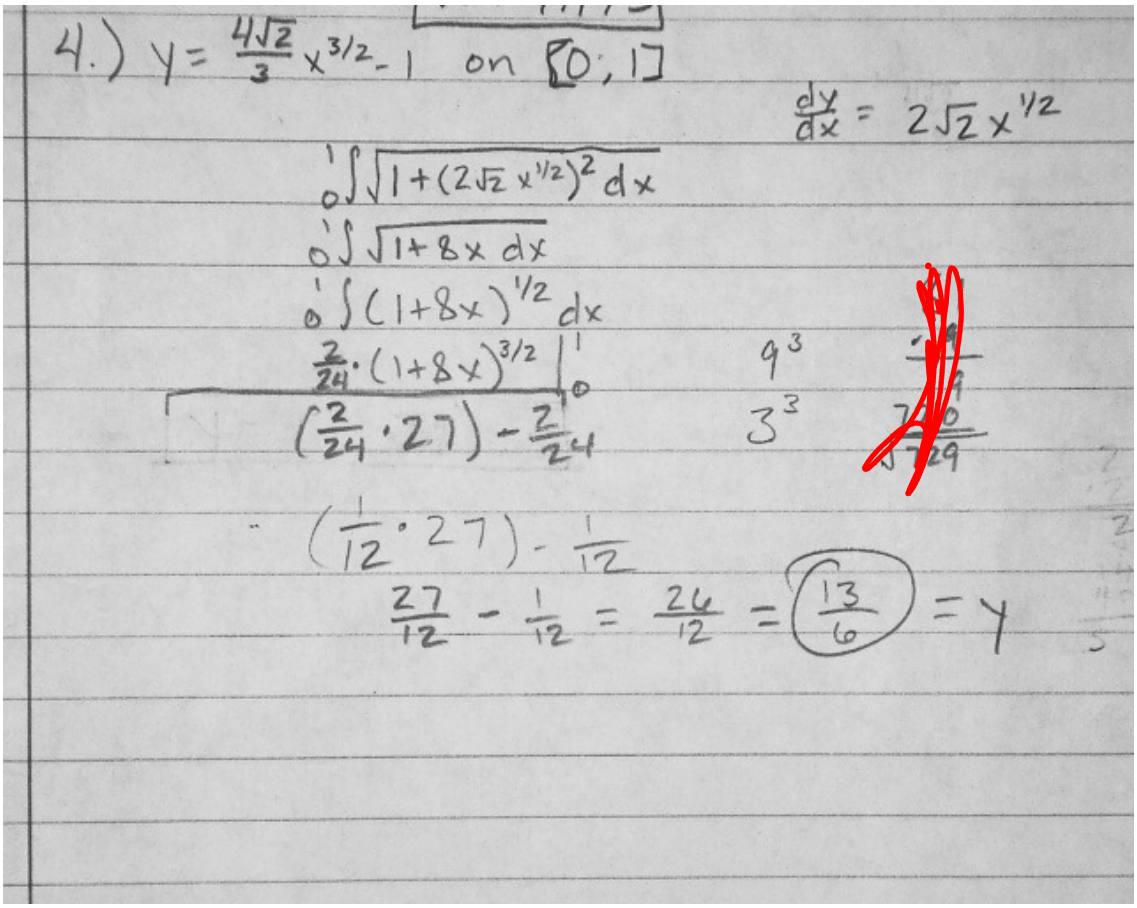
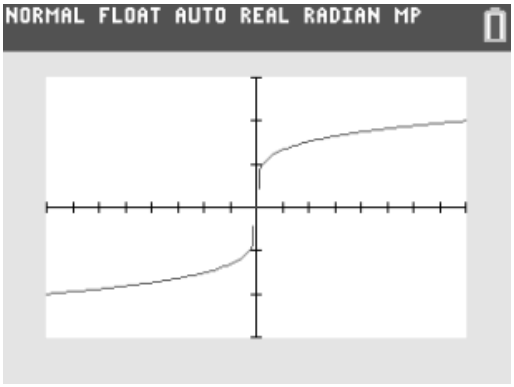
Ex2. Find the exact length of the curve (no GC)

$$y = \frac{4\sqrt{2}}{3}x^{3/2} - 1 \quad \text{on} \quad 0 \le x \le 1$$



A Vertical Tangent

Ex3. $y = \sqrt[5]{x}$ on $-32 \le x \le 32$



$x = y^5$

$\frac{dx}{dy} = 5y^4$

$-32 \le y^5 \le 32$

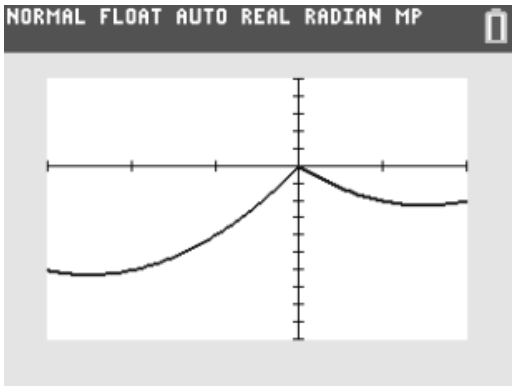
$-2 \le y \le 2$

$\int_{-2}^{2} \sqrt{1+(5y^4)^2}\,dy$

65.339

Corners and Cusps

Ex3. $y = x^2 - 4|x| + x$ on $-3 \le x \le 2$



split

$-3 \le x < 0$

$y = x^2 - 4(-x) + x$

$y = x^2 + 4x + x$

$y = x^2 + 5x$

$0 < x \le 2$

$y = x^2 - 4x + x$

$y = x^2 - 3x$

$$\int_{-3}^{0} \sqrt{1 + (2x+5)^2}\,dx + \int_{0}^{2} \sqrt{1 + (2x-3)^2}\,dx$$

10.926

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Homework

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